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Flavour-Changing-Neutral-Current Processes in B Physics

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Recent progress in flavour-changing-neutral-current (FCNC) processes is reviewed with particular emphasis on the role of QCD. Standard model (SM) is consistent with data. In particular, the rate and final-state distributions in radiative rare B decays, $B \rightarrow X_s + \gamma$, are well accounted for in the SM in terms of the short-distance contribution. The CLEO measurement of the inclusive rate $\mathcal{B}(B \rightarrow X_s \gamma) = (2.32 \pm 0.67) \times 10^{-4}$ yields $|V_{ts}|/|V_{cb}| = 0.85 \pm 0.23$, in agreement with unity expected from the unitarity of the CKM matrix. Likewise, most QCD-based theoretical estimates, using short-distance dominance, yield for the exclusive-to-inclusive decay rate ratio $\mathcal{B}(B \rightarrow K^* + \gamma)/\mathcal{B}(B \rightarrow X_s + \gamma) = 0.1 - 0.2$, in agreement with the data giving 0.19 ± 0.1 for the same quantity. Along the same lines, the present measurements of the branching ratio $\mathcal{B}(B \rightarrow K^* + \gamma)$ and upper limits on the branching ratios $\mathcal{B}(B \rightarrow \rho + \gamma)$ and $\mathcal{B}(B \rightarrow \omega + \gamma)$ can be interpreted in terms of an upper limit on the CKM matrix element ratio, yielding $|V_{td}|/|V_{ts}| \leq 0.75$. This bound is less restrictive than the unitarity bounds and the ones that follow from the lower limit on the $B^0 - \bar{B}^0$ mixing ratio $\Delta M_s/\Delta M_d > 11.3$, reported recently by the ALEPH collaboration. The potential of rare B decays in searching for physics beyond the standard model is also reviewed with emphasis on the decays $B \rightarrow X_s + \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$.

1. Introduction

Experimental evidence for the FCNC processes in B decays is at present based on the following quantities:

i) Exclusive radiative decay $B \rightarrow K^* + \gamma$ having a branching ratio [1]:

$$\mathcal{B}(B \rightarrow K^* + \gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}; \quad (1)$$

ii) the measurement of the inclusive photon energy spectrum in the decay $B \rightarrow X_s + \gamma$, yielding a branching ratio [2]:

$$\begin{aligned} \mathcal{B}(B \rightarrow X_s + \gamma) \\ = (2.32 \pm 0.51 \pm 0.32 \pm 0.20) \times 10^{-4}; \end{aligned} \quad (2)$$

and

iii) $B^0 - \bar{B}^0$ oscillations, which can be summarized in terms of the mass difference in the two neutral B meson systems [3]:

$$\begin{aligned} \Delta M_d &= 0.500 \pm 0.033 \text{ (ps)}^{-1} \\ \Delta M_s &> 9.0 \text{ (ps)}^{-1}, \end{aligned} \quad (3)$$

giving $\Delta M_s/\Delta M_d \geq 11.3$ (at 95% C.L.).

In SM [4], the $\Delta B = 1$, $\Delta Q = 0$ FCNC processes $b \rightarrow s + (\gamma, Z, \text{gluon})$, $b \rightarrow d + (\gamma, Z, \text{gluon})$,

and the $\Delta B = 2$, $\Delta Q = 0$ processes, $(\bar{b}s) \rightarrow (b\bar{s})$ and $(\bar{b}d) \rightarrow (b\bar{d})$, which dominantly contribute to the above transitions, are forbidden at the tree level and are governed by the (loop-induced) GIM amplitudes [5]. By virtue of this, the transition probabilities in FCNC decays are good measures of the quark mass differences appearing in the loop and the corresponding CKM matrix elements [6]. In particular, these measurements provide valuable information on the properties of the top quark, which gives dominant contributions in these transitions. As the top-quark mass is now measured ($m_t = 174 \pm 10_{-12}^{+13}$ GeV [7]), the above measurements can be used to provide information on the CKM matrix elements V_{it} ; $i = d, s, b$.

QCD plays a central role in quantifying these transitions. The GIM amplitudes are renormalized by QCD corrections and their calculations in radiative B decays [8,9], and $B^0 - \bar{B}^0$ mixings [10] have received quite some attention. Likewise, final-state distributions in the radiative decays $B \rightarrow X_s + \gamma$ and $B \rightarrow X_d + \gamma$ have been calculated in [11–13], including leading-order real and virtual gluon bremsstrahlung contributions in b -quark decays and B -meson wave-function effects, modelled after inclusive semileptonic de-

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cays $B \rightarrow X\ell\nu_\ell$ [14,15]. The exclusive decay $B \rightarrow K^* + \gamma$ has also been studied quite extensively using potential models [16], QCD sum rules of the older [17] and modern vintage [18–20], and lattice QCD-based methods [21].

The inclusive and exclusive decays $B \rightarrow X_s + \gamma$ and $B \rightarrow K^* + \gamma$ provide a calibration of the electromagnetic penguins in B decays. A crucial issue here is the relative importance of the long-distance effects due to the on-shell $u\bar{u}$ and $c\bar{c}$ intermediate states. It is easy to see that the former are suppressed by the CKM factors; in addition a dynamical suppression of the long-distance amplitudes is anticipated. This is in part due to the experimentally established suppression of the transverse helicity states in the decays $B \rightarrow V_1 V_2$, which using vector meson dominance contribute to the decays $B \rightarrow V\gamma$, and in part due to the suppression of the $\gamma - V$ vertex, as one extrapolates from the vector meson mass ($q^2 = m_V^2$), where such vertices are usually extracted from data, to the photon on-shell condition ($q^2 = 0$). These general expectations are borne out by a careful recent study [22], yielding

$$\left| \frac{\mathcal{A}_{B \rightarrow K^* + \gamma}^{VMD}}{\mathcal{A}_{B \rightarrow K^* + \gamma}^{exp}} \right| \leq 0.1, \quad (4)$$

which implies at most (10–20)% long-distance contribution in the decay rate for $B \rightarrow K^* + \gamma$. This goes hand-in-hand with the present data on the inclusive and exclusive radiative B decays (in branching ratios and distributions), which are entirely consistent with the dominance of the short-distance contributions, as discussed below.

There exists an overriding theoretical interest in measuring the CKM-suppressed B decays, $B \rightarrow X_d + \gamma$ and $B \rightarrow X_d + \ell^+ \ell^-$, in order to obtain independent constraints on the poorly known CKM matrix element V_{td} . These decays require high B meson statistics, which would be available in high luminosity B factories, HERA-B and LHC, and a good control on the dominant backgrounds from the CKM-allowed penguin decays $B \rightarrow X_s + \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$. Some exclusive decays such as $B_d \rightarrow \omega + \gamma$, $(B_d, B_u) \rightarrow \rho + \gamma$, $B_s \rightarrow K^* + \gamma$, and the FCNC semileptonic decays $B_u \rightarrow (\pi, \rho) + \ell^+ \ell^-$, $B_d \rightarrow (\pi, \omega, \rho) + \ell^+ \ell^-$ and $B_s \rightarrow (K, K^*) + \ell^+ \ell^-$ could do just as well. Their

measurements can be combined with the corresponding CKM-allowed exclusive decay modes $(B_d, B_u) \rightarrow K^* + \gamma$, $B_s \rightarrow \phi + \gamma$, $(B_d, B_u) \rightarrow (K, K^*) + \ell^+ \ell^-$ and $B_s \rightarrow \phi + \ell^+ \ell^-$, emerging from the decays $B \rightarrow X_s + \gamma$ and $B \rightarrow X_s + \ell^+ \ell^-$, to determine the CKM matrix element ratio $|V_{td}|/|V_{ts}|$ [12,19,20,23]. In the course of extracting this ratio due attention has to be paid to the intermediate $u\bar{u}$ and $c\bar{c}$ states [12]. In contrast to the CKM-allowed decays $B \rightarrow X_s + \gamma$, where the intermediate $u\bar{u}$ state is CKM-suppressed, in the decays $B \rightarrow X_d + \gamma$ the contributions of this and the $c\bar{c}$ state are of the same order of magnitude as far as the CKM factors are concerned. The dynamical suppression, discussed in the case of the decay $B \rightarrow K^* + \gamma$ applies for the decays $B \rightarrow (\rho, \omega) + \gamma$ as well; the numerical extent of this suppression will be reliably estimated as the CKM-suppressed non-leptonic decays $B \rightarrow (\rho, \omega)V$ are measured together with the relative contributions of the longitudinal and transverse helicity states.

Preliminary upper limits on the ratio $|V_{td}|/|V_{ts}|$ have been obtained based on the experimental upper limits on $B \rightarrow (\rho, \omega) + \gamma$ [24], assuming dominance of the short-distance contribution. In this context, possible dilution of such determinations due to the long-distance effects is emphasized in [25]. Judging on the test-case study of the decay $B \rightarrow K^* + \gamma$, in which the long-distance contribution from [22] is indicated in eq. (4), it is reasonable to expect that the corresponding contributions in the decays $B \rightarrow (\rho, \omega) + \gamma$ are not dominant either. Fortunately, the relative contribution of the long-distance effects in radiative decays will be better constrained from experiments when more precise data on neutral and charged B -meson decays become available.

Apart from testing the flavour sector of the SM precisely, FCNC processes provide potentially very promising search strategies for physics beyond the SM. In this context the decays $B \rightarrow X_s + \gamma$ and $B \rightarrow K^* + \gamma$ have been studied extensively. It has been argued that in some extensions of the SM, the constraints imposed on the parameters of these models by the radiative B decays are stringent and competitive with others derived from direct and indirect searches of non-

SM effects [26]. The decay rates and distributions in $B \rightarrow X_s + \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$ would allow the extraction of the magnitude and sign of the Wilson coefficients of the magnetic moment and the four-fermion operators involving the FCNC vertices $bs\gamma$ and $bs\ell^+\ell^-$ [27]. Likewise, exclusive decays $B \rightarrow (K, K^*)\ell^+\ell^-$ provide useful avenues in searching for non-SM effects [28].

This report is organized as follows. In section 2, we discuss the QCD-improved effective Hamiltonian approach for rare B decays $B \rightarrow X_s + \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$ in the standard model. Estimates of the inclusive decay rate $\mathcal{B}(B \rightarrow X_s + \gamma)$, taking into account the known QCD corrections, are reviewed and compared with the CLEO data to extract the CKM matrix element ratio $|V_{ts}|/|V_{cb}|$. Final-state distributions in the inclusive decay $B \rightarrow X_s + \gamma$ are discussed in section 3 and theoretical predictions [11] calculated in the SM are compared with the CLEO data. Inclusive radiative decays $B \rightarrow X_d + \gamma$ are discussed in section 4 and the prospects of determining the CKM matrix elements are reviewed. Estimates of the branching ratio $\mathcal{B}(B \rightarrow V + \gamma)$, with $V = K^*, \rho, \omega$, and the resulting bounds on the CKM matrix element ratio $|V_{td}|/|V_{ts}|$ are discussed in section 5. In section 6, the implications of the $B^0 - \bar{B}^0$ induced mass differences ΔM_d and ΔM_s for the parameters of the CKM matrix are discussed and the resulting constraints are compared with the profile of the CKM unitarity triangle, estimated in citeAL94. In section 7, we discuss a model-independent analysis of the FCNC decays $B \rightarrow X_s + \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$ showing some representative distributions concerning the dileptons in the latter case, illustrating the search strategies for new physics when data on the FCNC semileptonic decays become available. The constraints on the relevant Wilson coefficients from the present and impending experiments, worked out in the context of the minimal supersymmetric models (MSSM) in [27], are discussed in section 8.

2. The effective Hamiltonian for $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

The effective Hamiltonian in the SM describing the decays $b \rightarrow sX$, where X stands for a $\gamma, \ell^+ \ell^-$, gluon and a charm quark-antiquark pair, can be brought to the following expression

$$H_{eff}(b \rightarrow s) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu), \quad (5)$$

where the unitarity of the CKM matrix has been used in the form $\sum_{i=1,\dots,3} \lambda_i = 0$ with $\lambda_i = V_{ib} V_{is}^*$, and the terms proportional to λ_u have been neglected, since $\lambda_u \ll \lambda_c, \lambda_t$. The operator basis in $H_{eff}(b \rightarrow s)$ is restricted to dimension-6 operators and is chosen to be

$$\mathcal{O}_1 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha})(\bar{c}_{L\beta} \gamma_\mu c_{L\beta}) \quad (6)$$

$$\mathcal{O}_2 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\beta})(\bar{c}_{L\beta} \gamma_\mu c_{L\alpha}) \quad (7)$$

$$\mathcal{O}_3 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta} \gamma_\mu q_{L\beta}) \quad (8)$$

$$\mathcal{O}_4 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta} \gamma_\mu q_{L\alpha}) \quad (9)$$

$$\mathcal{O}_5 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta} \gamma_\mu q_{R\beta}) \quad (10)$$

$$\mathcal{O}_6 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta} \gamma_\mu q_{R\alpha}) \quad (11)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}_{L\alpha} \sigma_{\mu\nu} b_{R\alpha}) F^{\mu\nu} \quad (12)$$

$$\mathcal{O}_8 = \frac{g}{16\pi^2} m_b (\bar{s}_{L\alpha} T_{\alpha\beta}^a \sigma_{\mu\nu} b_{R\alpha}) G^{a\mu\nu} \quad (13)$$

$$\mathcal{O}_9 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha})(\bar{\ell} \gamma_\mu \ell) \quad (14)$$

$$\mathcal{O}_{10} = (\bar{s}_{L\alpha} \gamma_\mu b_{L\alpha})(\bar{\ell} \gamma_\mu \gamma_5 \ell) \quad (15)$$

$$(16)$$

with

$$q_L = \frac{1 - \gamma_5}{2} q \text{ and } q_R = \frac{1 + \gamma_5}{2} q. \quad (17)$$

Additional operators involving the mass of the s -quark are left out, as their contributions are small.

The procedure for the computation of the Wilson coefficients is by now standard. They are obtained at a large scale $\mu^2 = M_W^2$ in the SM or extensions of it (in general at $\mu^2 = m_Y^2$, where

m_Y stands generically for the mass of the heavy degrees of freedom). These coefficients define the matching conditions at the higher scale [30]. Then the renormalization group (RG) equations are used to scale down these coefficients to the scale that is typical for B -hadron decays, namely $\mu = O(m_b)$. In this way large logarithms of the form $\alpha_s \ln M_W^2$ are shifted from the matrix element to the Wilson coefficients. Following this prescription, one solves the RG equation:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] C_i \left(\frac{M_W^2}{\mu^2}, g \right) = \hat{\gamma}_{ji}(g) C_j \left(\frac{M_W^2}{\mu^2}, g \right), \quad (18)$$

where the perturbative beta function $\beta(g)$ can be seen in [31] and $\hat{\gamma}(g)$ is a (10×10) -dimensional anomalous-dimension matrix which, to leading logarithmic accuracy, has been derived in [9] (for earlier approximate results see [8]). This matrix and the analytic solutions of the RGE, which sum up the leading log terms of the form $(\alpha_s \ln M_W^2)^n$, can be seen in [27].

Starting from the effective Hamiltonian as given above, one finds that only one operator, namely \mathcal{O}_7 , contributes at tree level in the decay $b \rightarrow s\gamma$. Including QCD corrections brings to the fore other operators, and the QCD renormalization effects in the decay $b \rightarrow s\gamma$ can be expressed in terms of the renormalized Wilson coefficient $C_7(\mu)$. For a completely quantitative estimate of the inclusive decay rate for $B \rightarrow X_s + \gamma$ in the SM, one needs to know the coefficients $C_i(\mu)$ in the next-to-leading order and the matrix elements resulting from the one-loop QCD corrections to the matching conditions [32]. In addition, the final-state (photon energy and hadron mass) distributions in $B \rightarrow X_s + \gamma$ require calculation of the gluon bremsstrahlung and virtual corrections in the next-to-leading order as well. As the inclusive photon energy spectrum in the decay $B \rightarrow X_s + \gamma$ has now been measured and the experimental decay rate is obtained by integrating over this spectrum, a knowledge of the theoretical spectrum in the inclusive process $B \rightarrow X_s + \gamma$ is *mandatory* to estimate the decay rate.

Since the theoretical accuracy required for a reliable estimate of the decay rate $\mathcal{B}(B \rightarrow X_s + \gamma)$

is not at hand, due to the lack of the complete QCD corrections in the next-to-leading order, we will vary the scale parameter μ in the range $m_b/2 \leq \mu \leq 2m_b$ and use the resulting values of $C_i(\mu)$ and $\alpha_s(\mu)$ to hedge the present theoretical uncertainty, as advocated in [33]. The so-obtained effective Wilson coefficients are given numerically in Table 1, where we have used for illustration $\Lambda_{\text{QCD}} = 225$ MeV for five flavours and $m_t(\mu) = 165$ GeV, corresponding to the running top-quark mass, which is typically 9 GeV smaller than the pole mass. A remark concerning C_9 is in order. The effective coefficient which enters in the decay distributions in $B \rightarrow X_s \ell^+ \ell^-$ contains C_9 and a piece which comes from the one-loop matrix elements of the operators $O_1 \dots O_6$. This piece, ΔC_9 depends on the regularization scheme, which is chosen to evaluate these matrix elements in the leading-order approximation, as pointed out by Grinstein et al. in [34]. For the present top quark mass, this introduces a dependence of at most $\pm 20\%$ in the rates for $B \rightarrow X_s \ell^+ \ell^-$. This scheme-dependence can be reduced by including next-to-leading order terms in an effective C_9 [35]. However, for the present discussion this refinement is not crucial, much the same way that the other leading order results being discussed are also expected to be renormalized with improved theory.

It has become customary to express the branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$ in terms of the inclusive semileptonic branching ratio $\mathcal{B}(B \rightarrow X \ell \nu_\ell)$, which is well measured. The major uncertainty in the inclusive radiative and semileptonic decay rates, namely the dependence on the kinematic factor m_b^5 , cancels in the numerator and denominator. Also, the power corrections, calculated in the context of the heavy quark effective theory (HQET), are absent in the ratio of the inclusive rates up to and including the $(1/m_b^2)$ terms [36]. Finally, the model dependence in the ratio of the rates due to the wave function of the B meson is also minimal [11]. Therefore, one has:

$$\mathcal{B}(B \rightarrow X_s + \gamma) = 6 \frac{\alpha}{\pi} \frac{|\lambda_t|^2}{|V_{cb}|^2} \times \frac{|C_7(\mu)|^2 K(\mu)}{f(m_c/m_b)[1 - (2\alpha_s)/(3\pi)h(m_c/m_b)]} \times \mathcal{B}_{sl}, \quad (19)$$

Table 1: Values for the Wilson coefficients $C_i(\mu)$ at the scale $\mu = m_W$ (“matching conditions”) and at three other scales, $\mu = 10.0$ GeV, $\mu = 5.0$ GeV and $\mu = 2.5$ GeV, evaluated with one-loop β -function and the leading-order anomalous-dimension matrix, with $\overline{m}_t = 165$ GeV and $\Lambda_{\text{QCD}} = 225$ MeV.

$C_i(\mu)$	$\mu = m_W$	$\mu = 10.0$	$\mu = 5.0$	$\mu = 2.5$
C_1	0.0	0.158	0.229	0.318
C_2	-1.0	-1.063	-1.097	-1.145
C_3	0.0	-0.016	-0.021	-0.031
C_4	0.0	0.028	0.039	0.054
C_5	0.0	-0.005	-0.007	-0.009
C_6	0.0	0.019	0.003	0.004
C_7	0.188	0.269	0.30	0.336
C_8	0.093	0.13	0.143	0.157
C_9	-2.04	-2.0	-1.99	-1.98
C_{10}	4.56	4.56	4.56	4.56

where

$$f(r) = 1 - 8r^2 + 8r^6 - r^8 - 24r^4 \ln(r)$$

is the phase-space function for $\Gamma(b \rightarrow c + \ell \nu_\ell)$ and \mathcal{B}_{sl} is the semileptonic branching fraction, whose present average from $\Upsilon(4S)$ decays is $\mathcal{B}_{sl} = (10.29 \pm 0.06 \pm 0.27)\%$ [37]. The function $h(r)$ accounts for the $O(\alpha_s)$ QCD corrections to the semileptonic decay and can be found, for example, in ref. [14]. It is a slowly varying function of r ; for a typical quark-mass ratio of $r = 0.35 \pm 0.05$ it has the value $h(r) = 2.37 \mp 0.13$. The contributions from the decays $b \rightarrow u \ell \nu_\ell$ have been neglected in the denominator in eq. (19) since they are numerically inessential as $|V_{ub}| \ll |V_{cb}|$. Also to avoid problems with non-perturbative (renormalon) effects, the expansion parameter is taken to be the running mass $m_b(\mu)$ and the kinematic mass of the b quark is set equal to the running mass $m_b = m_b(\mu)$, as advocated in [38]. These definitions have a well-defined meaning in the NLO calculations. In anticipation, one may use these NLO expressions already in the lowest order.

The inclusive decay width for $B \rightarrow X_s + \gamma$ is dominantly contributed by the magnetic-moment operator \mathcal{O}_7 , hence the rationale of factoring out its coefficient in the expression for $\mathcal{B}(B \rightarrow X_s + \gamma)$ in Eq. (20). Including $O(\alpha_s)$ corrections to the matrix elements brings in other operators, in particular O_2 and O_8 . The effect of these additional terms can be expressed in terms of the function $K(\mu)$ in the integrated rate, which lumps together the effects of the bremsstrahlung corrections in the inclusive decay rate. With the present theoretical accuracy, which takes into account only leading effects, one finds $0.79 \leq K(\mu) \leq 0.86$ for $m_b/2 \leq \mu \leq 2m_b$ (see the last paper cited in [11]). Taking into account the running top-quark mass in the range $m_t(\mu) = 165 \pm 16$ GeV and the μ dependence of the Wilson coefficients, given in Table 1, gives a branching ratio:

$$\mathcal{B}(B \rightarrow X_s + \gamma) = (3.2 \pm 0.75) \times 10^{-4}, \quad (20)$$

to be compared with the result of the inclusive measurement by the CLEO collaboration [2]:

$$\mathcal{B}(B \rightarrow X_s + \gamma) = (2.32 \pm 0.67) \times 10^{-4}. \quad (21)$$

The SM estimates are in agreement with the experimental measurement within the stated errors. This can be interpreted as dominance of the short-distance contribution in the electromagnetic decays $B \rightarrow X_s + \gamma$, although more precise data and theoretical control are required for completely quantitative conclusions. A complete NLO calculation for this decay rate is lacking and very much needed. Further theoretical studies of the long-distance contributions, likewise, are required. A measure of the importance of long-distance effects is the difference in the inclusive radiative decay rates involving charged and neutral B mesons, $\Gamma(B^0 \rightarrow X_s^0 \gamma) - \Gamma(B^\pm \rightarrow X_s^\pm \gamma)$, since one expects the long-distance effects in the B^0 decays to be colour-suppressed compared to the B^\pm decays; the (penguin) short-distance contribution is however identical in the charged and neutral radiative B decays. We shall assume penguin dominance of the inclusive rate $\mathcal{B}(B \rightarrow X_s + \gamma)$ in which case the CLEO measurement can be used to put the following (90% C.L.) bound on $C_7(\mu)$:

$$0.19 \leq |C_7(\mu)| \leq 0.32. \quad (22)$$

Comparing with the values of $C_7(\mu)$ given in Table 1, we note that the experimental lower bound coincides with $C_7(m_W)$ while the corresponding upper bound is close to $C_7(m_b/2)$! This reflects the present intrinsic theoretical uncertainty and the experimental error, which are both large. This prevents from drawing very sharp conclusions. However, within the present uncertainty, one can put bounds on the non-SM contributions in $B \rightarrow X_s + \gamma$ [26] and $B \rightarrow X_s \ell^+ \ell^-$ [27], which are quite interesting.

In the SM context, the best use of the measurement of $\mathcal{B}(B \rightarrow X_s + \gamma)$, in our opinion, lies in the determination of the CKM matrix element ratio $\lambda_t/|V_{cb}|^2$. The unitarity constraint $\sum_i \lambda_i = 0$ states that $(\lambda_t/|V_{cb}|)^2 \simeq (\lambda_c/|V_{cb}|)^2 = |V_{cs}|^2$. Since the CKM matrix element $|V_{cs}|$ is known from independent measurements, such as charmed hadron decay $D \rightarrow \bar{K} e^+ \nu_e$, giving $V_{cs} = 1.01 \pm 0.18$ [31], the electromagnetic penguin decays $B \rightarrow X_s + \gamma$ provide another independent measurement of this quantity. The dependence of the branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$ on the CKM matrix element ratio $|V_{ts}|^2/|V_{cb}|^2$ is shown in Fig. 1³. The two curves starting from the origin delimit the SM expectations with the pole top-quark mass m_t and μ varied in the range $158 \text{ GeV} \leq m_t \leq 190 \text{ GeV}$ and $2.5 \text{ GeV} \leq \mu \leq 10.0 \text{ GeV}$. For the CLEO measurement $\mathcal{B}(B \rightarrow X_s + \gamma) = (2.32 \pm 0.67) \times 10^{-4}$, we get [39]:

$$\frac{|V_{ts}|}{|V_{cb}|} = 0.85 \pm 0.23. \quad (23)$$

This result also implies $|V_{cs}| = 0.85 \pm 0.23$, which agrees well with the value of $|V_{cs}|$ from D_{e3} decays quoted above. We note that the ratio $|V_{ts}|/|V_{cb}|$ is expected to be close to unity from the CKM unitarity considerations [31], and the determination of this ratio from inclusive radiative B decays is entirely consistent with that. We also note that an extraction of $|V_{ts}|$ from the exclusive decays $B \rightarrow K^* + \gamma$ and $D \rightarrow K^* + \ell \nu_\ell$ using heavy quark symmetry has also been reported in the literature, yielding: $|V_{ts}| = 0.035 \pm 0.01(\text{expt.}) \pm$

³Since $|V_{tb}| \simeq 1$ to a very good approximation, we have set $\lambda_t = |V_{ts}|$.

Figure 1. Inclusive branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$ in the SM as a function of the CKM-matrix element ratio $(|V_{ts}|/|V_{cb}|)^2$, with the pole top-quark mass in the range $158 \text{ GeV} \leq m_t \leq 190 \text{ GeV}$ and the QCD-scale parameter μ in the range $2.5 \text{ GeV} \leq \mu \leq 10 \text{ GeV}$. The experimental value $\mathcal{B}(B \rightarrow X_s + \gamma) = (2.32 \pm 0.67) \times 10^{-4}$ from CLEO measurements is also shown. (From [39].)

$0.011(\text{theory})$ [40]. This can be converted into a ratio $|V_{ts}|/|V_{cb}| = 0.90 \pm 0.43$, adding the errors in quadrature and using $|V_{cb}| = 0.039 \pm 0.06$ [29]. This determination is less precise than the one quoted above from the inclusive decay rate $\mathcal{B}(B \rightarrow X_s + \gamma)$.

3. Final-state distributions in the decay $B \rightarrow X_s + \gamma$

In order to calculate the final-state spectra in the decays $B \rightarrow X_s + \gamma$ two ingredients are needed:

- (i) Perturbative QCD contributions involving real and virtual gluon bremsstrahlung, and
- (ii) non-perturbative structure functions appear-

ing in the transition $B \rightarrow X_s + \gamma$.

Concerning point (i), there exist striking similarities in the end-point photon energy spectrum in the decays $B \rightarrow X_s + \gamma$ and the end-point lepton energy spectrum in $B \rightarrow X_u \ell \nu_\ell$, calculated in the QCD-improved parton model. In particular, ignoring m_u and m_s , the end-point photon energy spectrum [11] and the corresponding lepton energy spectrum [41] are given by the same expression, as far as the double log terms $\alpha_s \ln^2(1-x)$ (with x being the scaled photon or lepton energy) are concerned. These results, which are derived in the leading order in α_s are exponentiated leading to a universal all-order Sudakov form [42]. The next-to-leading order (single logs) and constant terms have also been calculated for the energy spectra. However, the structure functions are not yet calculable from first principles and have to be modelled. In the context of the HQET approach, it is possible to relate energy-weighted spectra in the inclusive radiative and semileptonic decays [43–45]. This can be used to improve the non-perturbative aspects in the semileptonic and radiative decays as data become more precise.

We discuss a particular model to implement the non-perturbative aspects in $B \rightarrow X_s + \gamma$. This is the Gaussian-distributed b -quark Fermi motion model [14,15], which was first introduced in the context of the inclusive semileptonic decays $B \rightarrow X \ell \nu_\ell$, slightly modified to implement the B -meson wave-function effects in radiative decays as well. This model provides an adequate phenomenological description of the experimental lepton energy spectrum in $B \rightarrow X \ell \nu_\ell$ [46]. Further support for it comes from theoretical considerations in the context of HQET [47].

In this model the b -quark in the B -hadron is given a non-zero momentum having a Gaussian distribution represented by an *a priori* free (but adjustable) parameter, p_F :

$$\phi(p) = \frac{4}{\sqrt{\pi} p_F^3} \exp\left(\frac{-p^2}{p_F^2}\right) \quad ; \quad p = |\vec{p}| \quad (24)$$

with the obvious normalization

$$\int_0^\infty dp p^2 \phi(p) = 1. \quad (25)$$

The energy-momentum constraint is imposed in

the form:

$$W^2 = m_B^2 + m_q^2 - 2m_B \sqrt{p^2 + m_q^2} \quad (26)$$

where m_B is the B -meson mass, W the effective momentum-dependent mass of the b -quark, and m_q the mass of the spectator quark in the B -meson, $B = \bar{b}q$. This model has two parameters: p_F and W , which are common for the inclusive semileptonic and radiative decays, in addition to the final-state quark mass. The dependence of the inclusive branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$ on p_F and m_s is negligible, but it is somewhat noticeable on W .

An estimate of the parameters p_F and W can be (and has been) obtained from fits of the inclusive lepton energy spectra in B -decays, varying p_F and the spectator quark mass m_q , which determines W , and the charm quark mass. Data do not constrain W tightly, and this parameter is allowed to vary in the range $W = 4.87 \pm 0.1$ GeV, which is favoured by the lattice-QCD and QCD-sum rule estimates [48]. For $W = 4.87$ GeV, the fit to the CLEO lepton energy spectrum yields: $p_F = 0.27 \pm 0.02$ GeV. A comparison of the photon energy spectrum resulting from this model with the CLEO data is shown in Fig. 2. The parameters in these fits correspond to $W = 4.87 \pm 0.1$ GeV, $p_F = 0.27 \pm 0.04$ GeV, in reasonable agreement with the corresponding values of these parameters resulting from the fits of the inclusive lepton energy spectrum, and $m_s = 0.5$ GeV. The agreement between theoretical spectra [11] and data is reasonably good. Based on this comparison, it is fair to say that the SM adequately accounts for the inclusive rate and decay distributions in the decays $B \rightarrow X_s + \gamma$. Obviously, there is room for improvements in both theory and experiment.

4. Inclusive radiative decays $B \rightarrow X_d + \gamma$ and determination of the CKM parameters

We briefly discuss here the CKM-suppressed inclusive radiative decays $B \rightarrow X_d + \gamma$ with the view of determining the CKM parameters, in particular the ratio $|V_{td}|/|V_{ts}|$. This was proposed in [12],

where the final-state spectra were also worked out. In close analogy with the $B \rightarrow X_s + \gamma$ case discussed earlier, the complete set of dimension-6 operators relevant for the processes $b \rightarrow d\gamma$, $d\gamma g$ and $b \rightarrow d\ell^+\ell^-$ can be written as:

$$H_{eff}(b \rightarrow d) = -\frac{4G_F}{\sqrt{2}} \xi_t \sum_{j=1}^{10} C_j(\mu) \hat{O}_j(\mu), \quad (27)$$

where $\xi_i = V_{ib} V_{id}^*$ for $i = t, c, u$. The CKM unitarity constraint for the $b \rightarrow d$ transitions reads as

$$\sum_i \xi_i = 0. \quad (28)$$

We note that all the CKM-angle-dependent quantities ξ_i ($i = t, c, u$) in eq. (28) are of the same order of magnitude, which is most easily seen using the Wolfenstein parametrization of the CKM matrix [49]. This is an important difference as compared to the effective Hamiltonian $\mathcal{H}_{eff}(b \rightarrow s)$ written earlier. This difference can be implemented by defining the operators \hat{O}_1 and \hat{O}_2 as follows [12]:

$$\begin{aligned} \hat{O}_1 &= -\frac{\xi_c}{\xi_t} (\bar{c}_{L\beta} \gamma^\mu b_{L\alpha}) (\bar{d}_{L\alpha} \gamma_\mu c_{L\beta}) \\ &\quad - \frac{\xi_u}{\xi_t} (\bar{u}_{L\beta} \gamma^\mu b_{L\alpha}) (\bar{d}_{L\alpha} \gamma_\mu u_{L\beta}), \\ \hat{O}_2 &= -\frac{\xi_c}{\xi_t} (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha}) (\bar{d}_{L\beta} \gamma_\mu c_{L\beta}) \\ &\quad - \frac{\xi_u}{\xi_t} (\bar{u}_{L\alpha} \gamma^\mu b_{L\alpha}) (\bar{d}_{L\beta} \gamma_\mu u_{L\beta}), \end{aligned} \quad (29)$$

Figure 2. Photon energy spectra from the B -meson reconstruction analysis in the inclusive decay $B \rightarrow X_s + \gamma$ as measured by the CLEO collaboration. The curve shown is the CLEO signal monte carlo incorporating the SM-based calculations [11]. (From [2].)

with the rest of the operators \hat{O}_j ; $j = 3 \dots 10$, defined like their counterparts O_j ; $j = 3 \dots 10$, with the obvious replacement $s \rightarrow d$. With this definition, the matching conditions and the solution of the RG equation become formally identical for the two operator basis O_j and \hat{O}_j . The difference lies in the matrix elements of the first two operators, with the CKM factors included explicitly. The discussion of the inclusive rate and the final-state distributions goes along very similar lines as for the decays $B \rightarrow X_s + \gamma$ with two important differences:

(i) The decay rate for $B \rightarrow X_d + \gamma$ is now a function of the CKM parameters ρ and η as the decay

rate no longer factorizes in an overall CKM factor, and

(ii) the photon and hadron energy spectra are also dependent on the CKM parameters.

We remark that observation (ii) allows, in principle, a measurement of CP violation in the CKM-suppressed radiative B decays $B \rightarrow X_d + \gamma$ through the measurement of the photon energy spectrum. The dependence of the decay rate and photon energy spectrum on the CKM parameters ρ and η has been evaluated in [12]. The branching ratio can be written as:

$$\begin{aligned} \mathcal{B}(B \rightarrow X_d + \gamma) &= D_1 |\xi_t|^2 \\ &\left\{ 1 - \frac{1 - \rho}{(1 - \rho)^2 + \eta^2} D_2 - \frac{\eta}{(1 - \rho)^2 + \eta^2} D_3 \right. \\ &\left. + \frac{D_4}{(1 - \rho)^2 + \eta^2} \right\}, \end{aligned} \quad (30)$$

where the CKM factors ξ_u and ξ_c , due to the intermediate $u\bar{u}$ and $c\bar{c}$ states have been expressed in terms of the Wolfenstein parameters A , λ , ρ and η :

$$\begin{aligned} \xi_u &= A \lambda^3 (\rho - i\eta), \\ \xi_c &= -A \lambda^3. \end{aligned} \quad (31)$$

The m_t -dependence of the coefficients D_i is rather weak, as shown in Table 2. To get the inclusive branching ratio the CKM parameters ρ and η have to be constrained from the unitarity fits. Taking the parameters from a recent fit, one gets [29]:

$$\begin{aligned} A &= 0.80 \pm 0.12, \\ \lambda &= 0.2208 \pm 0.0018, \\ \eta &= 0.33 \pm 0.09, \\ \rho &= -0.12^{+0.50}_{-0.16}, \end{aligned} \quad (32)$$

allowing a rather large uncertainty in the decay rate for $B \rightarrow X_d + \gamma$ - hence the interest in measuring it. Taking the central values of the parameters, one gets $\mathcal{B}(B \rightarrow X_d + \gamma) = 1.9 \times 10^{-5}$, which is approximately a factor 12 - 15 smaller than the branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$. It is to be remarked that the non-factorizing part in the CKM parameters, which is due to the intermediate $u\bar{u}$ and $c\bar{c}$ contributions, is typically 10% in the rate $\mathcal{B}(B \rightarrow X_d + \gamma)$. Thus, the so-called

Table 2: Values of the coefficients D_i entering in eq. (30) as a function of m_t (from ref. [12])

m_t (GeV)	D_1	D_2	D_3	D_4
160	0.20	0.17	0.04	0.11
180	0.21	0.17	0.03	0.10
200	0.22	0.16	0.03	0.10

long-distance contributions are not overwhelming in the inclusive decay rate.

An independent determination of the Wolfenstein parameters ρ and η will follow from the inclusive radiative branching ratios $\mathcal{B}(B \rightarrow X_s + \gamma)$ and $\mathcal{B}(B \rightarrow X_d + \gamma)$. That a number of unknowns, such as the top-quark mass and the QCD scale parameter, drop out from this ratio is obvious from the expressions given earlier. Likewise, non-perturbative effects are also minimal. The CLEO collaboration has already measured the inclusive photon spectrum in $B \rightarrow X_s + \gamma$. A measurement of the corresponding spectrum in the decays $B \rightarrow X_d + \gamma$ requires an excellent K/π separation. With this and $O(10^8)$ B mesons, expected to be collected at threshold B factories, one should be able to reconstruct several hundred $B \rightarrow X_d + \gamma$ events. This deserves attention of our experimental colleagues.

5. Estimates of $\mathcal{B}(B \rightarrow V + \gamma)$ and present constraints on the ratio $|V_{td}|/|V_{ts}|$

In this section we discuss the constraints on the CKM matrix element ratio $|V_{td}|/|V_{ts}|$ that would follow from the measurements of the exclusive radiative B decays $B \rightarrow V + \gamma$, with $V = K^*, \rho, \omega$. As already mentioned, this proposal is feasible only if the long-distance contributions in these decays, which do not depend on V_{td} or V_{ts} , are small. Encouraged with the estimates in the inclusive radiative decays, discussed in the previous section, and estimates of the same in $B \rightarrow K^* + \gamma$ [22], we shall continue assuming that this is indeed the case in both the CKM-allowed and CKM-suppressed exclusive radiative

B decays.

We review here the main results from a calculation of the form factors carried out in the context of the light-cone QCD sum rules [19]. While the form factor estimates given below are specific to this model, the results for the ratios, which are the main objects of interest in the determination of the CKM parameters, should be fairly representative, as borne out by similar independent calculations [20,23].

The exclusive decays $(B_u, B_d) \rightarrow (K^*, \rho) + \gamma$, $B_d \rightarrow \omega + \gamma$ and the corresponding B_s decay, $B_s \rightarrow (\phi, K^*) + \gamma$, involve the magnetic moment operator O_7 and the related one obtained by the obvious change $s \rightarrow d$. The transition form factors governing the radiative B decays $B \rightarrow V + \gamma$ can be generically defined as:

$$\begin{aligned} \langle V, \lambda | \frac{1}{2} \bar{\psi} \sigma_{\mu\nu} q^\nu b | B \rangle \\ = i \epsilon_{\mu\nu\rho\sigma} e_\nu^{(\lambda)} p_B^\rho p_V^\sigma F_1^{B \rightarrow V}(0). \end{aligned} \quad (33)$$

Here V is a vector meson with the polarization vector $e^{(\lambda)}$, $V = \rho, \omega, K^*$ or ϕ ; B is the generic B -meson B_u, B_d or B_s , and ψ stands for the field of a light u, d or s quark. The vectors p_B, p_V and $q = p_B - p_V$ correspond to the 4-momenta of the initial B -meson and the outgoing vector meson and photon, respectively. In (33) the QCD renormalization of the $\bar{\psi} \sigma_{\mu\nu} q^\nu b$ operator is implied, corresponding to the choice $\mu = O(m_b)$ in the effective Hamiltonian given earlier. With the above definition, the decay width $\Gamma(B \rightarrow K^* + \gamma)$ can be expressed as ($B = B_u$ or B_d):

$$\begin{aligned} \Gamma(B \rightarrow K^* \gamma) &= \frac{\alpha}{32\pi^4} G_F^2 |\lambda_t|^2 |F_1^{B \rightarrow K^*}(0)|^2 C_7(\mu)^2 \\ &\times (m_b^2 + m_s^2) \frac{(m_B^2 - m_{K^*}^2)^3}{m_B^3}. \end{aligned} \quad (34)$$

Likewise, for the decay width $\Gamma(B_s \rightarrow \phi + \gamma)$, we have:

$$\begin{aligned} \Gamma(B_s \rightarrow \phi + \gamma) &= \frac{\alpha}{32\pi^4} G_F^2 |\lambda_t|^2 |F_1^{B_s \rightarrow \phi}(0)|^2 C_7(\mu)^2 \\ &\times (m_b^2 + m_s^2) \frac{(m_{B_s}^2 - m_\phi^2)^3}{m_{B_s}^3}. \end{aligned} \quad (35)$$

A good quantity to test the model dependence of the exclusive decay form factors is the ratio of

the exclusive-to-inclusive radiative decay widths ($B_i = B_u$ or B_d):

$$R_i(K^*/X_s) \equiv \frac{\Gamma(B_i \rightarrow K^* + \gamma)}{\Gamma(B_i \rightarrow X_s + \gamma)}. \quad (36)$$

With the branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$ defined above, we have

$$\begin{aligned} R_i(K^*/X_s) &= \frac{(1 - m_{K^*}^2/m_B^2)^3}{(1 - m_s^2/m_b^2)^3} \\ &\times \frac{|F_1^{B_i \rightarrow K^*}(0)|^2}{K(\mu)} \frac{m_B^3}{m_b^3} \frac{1}{f(\lambda_1, \lambda_2)}. \end{aligned} \quad (37)$$

The function $K(\mu)$ is almost independent of m_t and has been estimated earlier; the factor $f(\lambda_1, \lambda_2) = 1 + (\lambda_1 - 9\lambda_2)/2m_b^2$ is due to power corrections to the inclusive radiative decay rate. The non-perturbative parameters have been estimated as $\lambda_1 = -0.15 \pm 0.15 \text{ GeV}^2$ and $\lambda_2 = 0.12 \pm 0.01 \text{ GeV}^2$ in the HQET approach [36]. The exclusive-to-inclusive ratio involving the B_s decays is defined analogously.

For the two-body decays $b \rightarrow s + \gamma$ and $b \rightarrow d + \gamma$, only the magnetic moment operator O_7 (and its analogue) contributes. QCD renormalizes the Wilson coefficient and brings in non-factorizing contributions in terms of the CKM parameters, as discussed earlier in the context of the inclusive decays $B \rightarrow X_d + \gamma$. In the approximation of dropping the $O((m_c/m_t)^2)$ and $O((m_u/m_t)^2)$ terms, the QCD-corrected amplitudes factorize in the CKM factors. This leads to relations among the exclusive decay rates, exemplified here by the decay rates for $(B_u, B_d) \rightarrow \rho + \gamma$ and $(B_u, B_d) \rightarrow K^* + \gamma$.

$$\begin{aligned} &\frac{\Gamma((B_u, B_d) \rightarrow \rho + \gamma)}{\Gamma((B_u, B_d) \rightarrow K^* + \gamma)} \\ &= \frac{|\xi_t|^2}{|\lambda_t|^2} \frac{|F_1^{B \rightarrow \rho}(0)|^2}{|F_1^{B \rightarrow K^*}(0)|^2} \Phi_{u,d}, \end{aligned} \quad (38)$$

where $\Phi_{u,d}$ is a phase-space factor defined as:

$$\Phi_{u,d} = \frac{(m_b^2 + m_d^2)}{(m_b^2 + m_s^2)} \frac{(m_{B_{u,d}}^2 - m_\rho^2)^3}{(m_{B_{u,d}}^2 - m_{K^*}^2)^3}. \quad (39)$$

The ratio (38) is independent of the top-quark mass as well as the scale parameter μ . It depends, of course, on the CKM-matrix elements,

apart from the ratio of form factors. The decay width $\Gamma(B_d \rightarrow \omega + \gamma)$ is expected to be equal to the decay width $\Gamma(B_d \rightarrow \rho + \gamma)$, apart from the minor difference in the phase-space factors Φ . This follows from the assumption that the quark wave functions for ω and ρ are described by the isoscalar and isovector combinations, $|\omega\rangle = 1/\sqrt{2}(\bar{u}u + \bar{d}d)$ and $|\rho\rangle = 1/\sqrt{2}(\bar{u}u - \bar{d}d)$, respectively. The decay width $\Gamma(B_s \rightarrow K^* + \gamma)$ can be related to the CKM-allowed decay width $\Gamma(B_d \rightarrow K^* + \gamma)$ by a relation similar to the one given in Eq. (38). However, one expects a substantial difference in the form factors $F_1^{B_d \rightarrow K^*}(0)$ and $F_1^{B_s \rightarrow K^*}(0)$, due to the exchange of the roles of s and d quarks in the wave function of the K^* in the two decay modes, as pointed out in [19].

The results of the transition form factors evaluated in the light-cone QCD sum-rule approach can be expressed as [19]:

$$\begin{aligned}\sqrt{2}F_1^{B_d \rightarrow K^{*0}\gamma} &= F_1^{B_u \rightarrow K^{*+}\gamma} = 0.32 \pm 0.05, \\ \sqrt{2}F_1^{B_d \rightarrow (\rho, \omega)\gamma} &= F_1^{B_u \rightarrow \rho^+\gamma} = 0.24 \pm 0.04, \\ F_1^{B_s \rightarrow \phi\gamma} &= 0.29 \pm 0.05, \\ F_1^{B_s \rightarrow K^*\gamma} &= 0.20 \pm 0.04,\end{aligned}\quad (40)$$

For the ratios of the form factors, which are needed to determine the ratios of the CKM matrix-elements, the following estimates have been reported in [19]:

$$\frac{F_1^{B_d \rightarrow (\rho, \omega)\gamma}}{F_1^{B_d \rightarrow K^*\gamma}} = 0.76 \pm 0.06, \quad (41)$$

$$\frac{F_1^{B_s \rightarrow K^* + \gamma}}{F_1^{B_s \rightarrow \phi + \gamma}} = 0.66 \pm 0.09, \quad (42)$$

and

$$\frac{F_1^{B_s \rightarrow K^* + \gamma}}{F_1^{B_d \rightarrow K^* + \gamma}} = 0.60 \pm 0.12. \quad (43)$$

The measurements of the CKM-suppressed radiative decays $(B_u, B_d) \rightarrow \rho + \gamma$, $B_d \rightarrow \omega + \gamma$ and $B_s \rightarrow K^* + \gamma$ could then be used to determine the CKM parameters. This is illustrated in Fig. 3, where the ratio of the branching ratios $\mathcal{B}(B_u \rightarrow \rho + \gamma)/\mathcal{B}(B \rightarrow K^* + \gamma)$ and $\mathcal{B}(B_s \rightarrow$

$K^* + \gamma)/\mathcal{B}(B \rightarrow K^* + \gamma)$ are plotted as a function of the CKM-matrix-element ratio $|V_{td}|/|V_{ts}|$. The present upper limits on the branching ratios $\mathcal{B}(B \rightarrow (\rho, \omega) + \gamma)$ have been combined with the measured branching ratio for $\mathcal{B}(B \rightarrow X_s + \gamma)$ and the above estimates of the form-factor ratios to put the following upper limit on the CKM-matrix-element ratio [24]:

$$\frac{|V_{td}|}{|V_{ts}|} \leq 0.75, \quad (44)$$

with a theoretical dispersion in the range 0.64 - 0.75, depending on the models used in estimates of the $SU(3)$ breaking in the form factors [19, 20, 23]. This bound is not yet competitive with the bounds on the corresponding ratio of the CKM-matrix elements obtained from the unitarity fits, which yields at 95% C.L. [29]:

$$0.11 \leq \frac{|V_{td}|}{|V_{ts}|} \leq 0.36, \quad (45)$$

Thus, with short-distance dominance, the expected rates for the CKM-suppressed radiative decays are an order of magnitude lower than the present experimental bounds. It should be possible to measure them in threshold B factories, which will also provide reliable estimates of the long-distance effects.

6. Constraints on the unitarity triangle from $B^0 - \bar{B}^0$ mixings

It is well appreciated that measurements of the weak-mixing induced mass differences ΔM_d and ΔM_s in the neutral B meson systems provide useful constraints on the CKM parameters. We review the status of such constraints.

The present world average of the $B_d^0 - \bar{B}_d^0$ mixing parameter $x_d \equiv \Delta M_d/\Gamma_d$ is $x_d = 0.76 \pm 0.06$ [3]. The precision on ΔM_d is now comparable to the one on the time-integrated quantity x_d . The LEP-average $\Delta M_d = 0.513 \pm 0.036$ (ps) $^{-1}$ has been combined with that derived from time-integrated measurements yielding the present world average [3]

$$\Delta M_d = 0.500 \pm 0.033 \text{ (ps)}^{-1}. \quad (46)$$

In SM, ΔM_d is calculated from the $B_d^0\text{-}\overline{B}_d^0$ box diagram, which is dominated by t -quark exchange:

$$\begin{aligned} \Delta M_d & \\ &= \frac{G_F^2}{6\pi^2} M_W^2 M_B (f_{B_d}^2 B_{B_d}) \hat{\eta}_B y_t f_2(y_t) |V_{td}^* V_{tb}|^2, \end{aligned} \quad (47)$$

where $|V_{td}^* V_{tb}|^2 = A^2 \lambda^6 [(1-\rho)^2 + \eta^2]$. Here, $\hat{\eta}_B$ is the QCD renormalization factor for which a value $\hat{\eta}_B = 0.55$ has been calculated in the \overline{MS} scheme [50]. Consistency requires that the top-quark mass be rescaled from its pole (mass) value of $m_t = 174 \pm 16$ GeV to the value $\overline{m}_t(m_t(\text{pole}))$ in the \overline{MS} scheme, which is typically about 9 GeV smaller. For the hadronic quantity $f_{B_d}^2 B_{B_d}$ we take ranges for f_{B_d} and \hat{B}_{B_d} , which are compatible with recent lattice QCD s [51] and QCD sum-rule results [52]:

$$\begin{aligned} f_{B_d} &= 180 \pm 50 \text{ MeV}, \\ \hat{B}_{B_d} &= 1.0 \pm 0.2. \end{aligned} \quad (48)$$

The $B_s^0\text{-}\overline{B}_s^0$ box diagram is also dominated by t -quark exchange, and the mass difference between the eigenstates ΔM_s is given by the expression:

$$\begin{aligned} \Delta M_s &= \\ &= \frac{G_F^2}{6\pi^2} M_W^2 M_{B_s} (f_{B_s}^2 B_{B_s}) \hat{\eta}_{B_s} y_t f_2(y_t) |V_{ts}^* V_{tb}|^2. \end{aligned} \quad (49)$$

A measurement of ΔM_s can be used to give an additional constraint on the unitarity triangle. Taking the ratio of ΔM_d and ΔM_s , one gets

$$\frac{\Delta M_s}{\Delta M_d} = \frac{\hat{\eta}_{B_s} M_{B_s} (f_{B_s}^2 B_{B_s})}{\hat{\eta}_{B_d} M_{B_d} (f_{B_d}^2 B_{B_d})} \left| \frac{V_{ts}}{V_{td}} \right|^2. \quad (50)$$

Figure 3. Ratio of the branching ratios $\mathcal{B}(B_u \rightarrow \rho + \gamma)/\mathcal{B}(B \rightarrow K^* + \gamma)$, and $\mathcal{B}(B_s \rightarrow K^* + \gamma)/\mathcal{B}(B \rightarrow K^* + \gamma)$, where $B = B_u$ or B_d , as a function of the CKM-matrix element ratio $|V_{td}|/|V_{ts}|$. The two lines correspond to the input parameters used in the QCD sum-rule approach in [19].

All dependence on the t -quark mass drops out, leaving the square of the ratio of CKM matrix elements, multiplied by a factor which reflects $SU(3)_{flavour}$ -breaking effects. Since we expect the QCD correction factor $\hat{\eta}_{B_s}$ to be equal to its B_d counterpart, the only real uncertainty in this factor is the ratio of hadronic matrix elements. In what follows, we take

$$\xi_s \equiv \frac{(f_{B_s} \sqrt{\hat{B}_{B_s}})}{(f_{B_d} \sqrt{\hat{B}_{B_d}})} = (1.16 \pm 0.1), \quad (51)$$

consistent with estimates from lattice QCD [51] and from QCD sum rules [52].

The ALEPH lower bound $\Delta M_s/\Delta M_d > 11.3$ at 95% C.L. [3] can thus be turned into a bound on the CKM parameter space (ρ, η) by choosing a value for the $SU(3)$ -breaking parameter ξ_s^2 . We assume three representative values: $\xi_s^2 = 1.1, 1.35$ and 1.6 , and display the resulting constraints in Fig. 4. The closed curve in this figure represents the 95% C.L. contour resulting from the CKM fits in [29]. This fit uses the parameters given in eq. (48) and $\hat{B}_K = 0.8 \pm 0.2$, which parametrizes the hadronic uncertainty in the analysis of ϵ_K , the CP-violating parameter in the kaon system. From this graph we see that the ALEPH bound on $\Delta M_d/\Delta M_s$ marginally restricts the allowed ρ - η region for small values of ξ_s^2 , but does not provide any useful bounds for larger values. Very similar conclusions are drawn if instead one uses a lower range $\hat{B}_K = 0.6 \pm 0.2$.

We now discuss the SM prediction for $x_s \equiv \Delta M_s/\Gamma_s$. The main uncertainty in x_s (or, equivalently, ΔM_s) is $f_{B_s}^2 B_{B_s}$. Using the determination of A from [29], $A = 0.80 \pm 0.12$, $\tau_{B_s} = 1.54 \pm 0.14$ ps and $\overline{m}_t = 165 \pm 16$ GeV, we obtain

$$x_s = (19.4 \pm 6.9) \frac{f_{B_s}^2 B_{B_s}}{(230 \text{ MeV})^2} . \quad (52)$$

The choice $f_{B_s} \sqrt{\hat{B}_{B_s}} = 230$ MeV corresponds to the central value given by the lattice-QCD estimates [51], and with this the CKM fit gives $x_s \simeq 20$ as the preferred value in the SM. Experiments at LEP and Tevatron and HERA-B will probe the lower half of the range in (52). Of course, an actual measurement of x_s or ΔM_s would be very helpful in constraining the CKM parameter space.

7. Towards a Model-Independent Analysis of the Decays $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

The determination of $|C_7(m_b)|$ from the inclusive branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$ is a prototype of the kind of analysis that one would like to be carried out for the rare B decays in general and for the semileptonic decays $B \rightarrow X_s \ell^+ \ell^-$, in particular. First steps towards a model-independent analysis of the FCNC electroweak rare B decays

Figure 4. Allowed region in ρ - η space, from a simultaneous fit to the experimental and theoretical quantities. Values of the non-perturbative parameters used in the fit are indicated on top of the figure. The solid line represents the region with $\chi^2 = \chi_{min}^2 + 6$ corresponding to the 95% C.L. region. The triangle shows the best fit. The constraints in ρ - η space from the ALEPH bound on ΔM_s are presented for three choices of the $SU(3)$ -breaking parameter: $\xi_s^2 = 1.1$ (dotted line), 1.35 (dashed line) and 1.6 (solid line). (From [29].)

involving these decay modes have recently been proposed in [27].

The main interest in rare B decays is to measure the effective FCNC vertices in order to test the SM and search for new physics. With some plausible assumptions these vertices can be parametrized through a limited number of effective parameters, which govern the rates and shapes (differential distributions) in the rare B decays $B \rightarrow X_s \gamma$, $B \rightarrow X_s \ell^+ \ell^-$ and $B_s \rightarrow \ell^+ \ell^-$. Since the coefficients C_1, \dots, C_6 determine the bulk properties of the B hadrons, such as the lifetime and the semileptonic branching ratio, and the SM estimates are in broad agreement with data, these coefficients can be fixed to their SM values. The search for physics beyond the SM in the radiative and FCNC semileptonic decays, therefore, can be carried out in terms of three effective parameters, $C_7(\mu)$, $C_9(\mu)$ and $C_{10}(\mu)$, characterizing the strength of the magnetic moment and two four-fermion operators $(\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell)$ and $(\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma^5 \ell)$. This can then be interpreted in a large class of models, of which the SM values are given in Table 1. The presence of non-SM physics may manifest itself by distorting the differential distributions in $B \rightarrow X_s \ell^+ \ell^-$. Some possible examples of such distortions have been worked out in [27], which we summarize below.

It is known that the Dalitz distribution in $B \rightarrow X_s \ell^+ \ell^-$ and the decay rate in $B \rightarrow X_s + \gamma$ are sensitive to the presence of new physics. While many experimental quantities can be studied and are of interest, we concentrate on the following:

- (i) Inclusive radiative rare decay branching ratio $\mathcal{B}(B \rightarrow X_s + \gamma)$;
- (ii) Invariant dilepton mass distributions in $B \rightarrow X_s \ell^+ \ell^-$;
- (iii) Forward-backward (FB) charge asymmetry $\mathcal{A}(\hat{s})$ in $B \rightarrow X_s \ell^+ \ell^-$.

The FB asymmetry $\mathcal{A}(\hat{s})$ is defined with respect to the angular variable $z \equiv \cos \theta$, where θ is the angle of the ℓ^+ with respect to the b -quark direction in the centre-of-mass system of the dilepton pair. It is obtained by integrating the doubly differential distribution $d^2\mathcal{B}/(dz d\hat{s})(B \rightarrow$

$X_s \ell^+ \ell^-)$ [34]:

$$\mathcal{A}(\hat{s}) \equiv \int_0^1 dz \frac{d^2\mathcal{B}}{dz d\hat{s}} - \int_{-1}^0 dz \frac{d^2\mathcal{B}}{dz d\hat{s}}, \quad (53)$$

where $\hat{s} = (p_1 + p_2)^2/m_b^2$ and p_1 and p_2 denote, respectively, the momenta of the ℓ^+ and ℓ^- .

The decay rate $\mathcal{B}(B \rightarrow X_s + \gamma)$ puts a bound on the absolute value of the coefficient $C_7(\mu)$. Fixing the overall sign in the SM (a convention), it is interesting to observe that both the positive and negative C_7 solutions are allowed, for example, in the MSSM as one scans over the allowed parameter space. However, the radiative B decay rate by itself is not able to distinguish between the solutions $C_7(\mu) > 0$ and the solutions $C_7(\mu) < 0$. The Dalitz distribution, in general, and the invariant dilepton mass distribution and the forward-backward asymmetry in $B \rightarrow X_s \ell^+ \ell^-$, in particular, are sensitive to the relative signs and magnitude of $C_7(\mu)$ and the other coefficients [34]. Using \mathcal{H}_{eff} , as given above, one obtains for the dilepton invariant mass distribution

$$\begin{aligned} \frac{d\mathcal{B}}{d\hat{s}} = & \mathcal{B}_{sl} \frac{\alpha^2}{4\pi^2} \frac{\lambda_t^2}{|V_{cb}|} \frac{\hat{w}(\hat{s})}{f(m_c/m_b)} \\ & \times \left[(|C_9 + Y(\hat{s})|^2 + C_{10}^2) \alpha_1(\hat{s}, \hat{m}_s) \right. \\ & \left. + \frac{4}{\hat{s}} C_7^2 \alpha_2(\hat{s}, \hat{m}_s) + 12\alpha_3(\hat{s}, \hat{m}_s) C_7(C_9 + \text{Re } Y(s)) \right], \end{aligned} \quad (54)$$

where the auxiliary functions α_i depend only on the kinematic variables and $Y(\hat{s})$ depends on the coefficients C_1, \dots, C_6 of the four-quark operators (see [27]). It also contains a scheme-dependent piece, in the leading log approximation, as already discussed. These coefficients are all assumed to have the SM values, with the implied improvements in the QCD framework taken into account as they become available.

The corresponding differential asymmetry as defined in (53) is

$$\begin{aligned} \mathcal{A}(\hat{s}) = & -\mathcal{B}_{sl} \frac{3\alpha^2}{8\pi^2} \frac{1}{f(m_c/m_b)} \hat{w}^2(\hat{s}) C_{10} \\ & \times [\hat{s}(C_9 + \text{Re } Y(\hat{s})) + 4C_7(1 + \hat{m}_s^2)]. \end{aligned} \quad (55)$$

As shown above, the asymmetry is directly proportional to C_{10} . The invariant mass dependence

Figure 5. The dependence of the invariant-mass spectrum on the Wilson coefficients. Solid line: SM. Long-dashed line: $C_7 \rightarrow -C_7$, with other coefficients retaining their SM values. Short-dashed line: The contribution of C_{10} only. Dotted line: $C_{10} = 0$, with other coefficients retaining their SM values. Dash-dotted line: same as for the dotted one, but with $C_7 = -0.3$. The vertical lines indicate the location of the J/Ψ and Ψ' resonances. (From [27].)

of the asymmetry can be combined with the dilepton mass spectrum distribution to determine all the three Wilson coefficients in sign and magnitude. In fig. 5 we plot the various contributions to the spectrum, for positive and for negative C_7 , to illustrate this point; various (assumed) contributions to $\mathcal{A}(s)$ are shown in fig. 6, again underlying the sensitivity of this measurement to non-SM contributions.

8. Model Predictions for the Wilson Coefficients in $\mathcal{H}_{eff}(b \rightarrow s)$

As an illustrative example of the non-SM physics the minimal supersymmetric standard model is perhaps the best motivated and quite instructive. In [27], estimates of the Wilson coefficients C_7 , C_9 , and C_{10} are worked out for this model taking into account the present experimental constraints and theoretical considerations; they are summarized here.

Once the gluino contributions (as well as the

Figure 6. The dependence of the differential FB asymmetry on the Wilson coefficients. Solid line: SM. Long-dashed line: $C_7 \rightarrow -C_7$, with the other parameters retaining their SM values. The vertical lines indicate the location of the J/Ψ and Ψ' resonances. (From [27].)

analogous ones from neutralino exchange) are neglected, the flavour violation in the supersymmetric models is completely specified by the familiar CKM matrix. The one-loop supersymmetric corrections to the Wilson coefficients C_7 , C_9 , and C_{10} are given by two classes of diagrams: charged-Higgs exchange and chargino exchange [53]. The charged-Higgs contribution is specified by two input parameters: the charged-Higgs mass (m_{H^\pm}) and the ratio of Higgs vacuum expectation values ($v_2/v_1 \equiv \tan \beta$). This contribution alone corresponds to the two-Higgs doublet model which, has also been considered in [27].

The chargino contribution is specified by six parameters. Three of them enter the 2×2 chargino mass matrix:

$$m_{\chi^\pm} = \begin{pmatrix} M & m_W \sqrt{2} \sin \beta \\ m_W \sqrt{2} \cos \beta & \mu \end{pmatrix}. \quad (56)$$

Following standard notations, we call $\tan \beta$ the ratio of vacuum expectation values, the same as appears also in the charged-Higgs sector, and M , μ the gaugino and higgsino mass parameters, subject to the constraint that the lightest-chargino mass satisfies the LEP bound, $m_{\chi^\pm}^+ > 45$ GeV.

The squark masses

$$m_{\tilde{q}_{\pm}}^2 = \tilde{m}^2 + m_q^2 \pm A\tilde{m}m_q \quad (57)$$

contain two additional free parameters besides the known mass of the corresponding quark m_q : a common supersymmetry-breaking mass \tilde{m} and the coefficient A . The last parameter included in the analysis in [27] is a common mass $m_{\tilde{l}}$ for sleptons, all taken to be degenerate in mass, with the constraint $m_{\tilde{l}} > 45$ GeV. Therefore the version of the MSSM being considered is defined in terms of seven free parameters.

The Wilson coefficients in the MSSM have been computed in [27] by varying the seven above-defined parameters in the experimentally allowed region. The results of this analysis are presented in fig. 7, which shows the regions of the C_9 - C_{10} plane allowed by possible choices of the MSSM parameters. The upper plot of fig. 7 corresponds to parameters that give rise to positive (same sign as in the SM) values of C_7 , consistent with experimental results on $B \rightarrow X_s + \gamma$ ($0.19 < C_7 < 0.32$), while the lower plot corresponds to values of C_7 with opposite sign ($-0.32 < C_7 < -0.19$). The contour resulting from improved experimental limits on supersymmetric particle masses, as can be expected from the Tevatron and LEP 200, are also shown in Fig. 7. They correspond to the (assumed) constraints $m_{H^+} > 150$ GeV, $m_{\tilde{l}}, m_{\chi^+}, m_{\tilde{l}} > 100$ GeV.

The regions shown in fig. 7 illustrate the typical trend of the supersymmetric corrections. If supersymmetric particles exist at low energies, we can expect values of C_9 and C_{10} , which are, respectively, smaller (negative) and larger than those predicted by the SM. This is the general feature, although the exact boundaries of the allowed regions depend on the particular model-dependent assumptions one prefers to use. However, the most interesting feature of supersymmetry is that solutions with both the positive and negative values of C_7 are possible and are still consistent with present data. Moreover, values of the other two coefficients C_9 and C_{10} sufficiently different from the SM are allowed, leading to measurable differences in the decay rates and distributions of $B \rightarrow X_s \ell^+ \ell^-$ and $B_s \rightarrow \ell^+ \ell^-$. The analysis in [27] suggests that rare B decays have a discov-

Figure 7. The region in the C_9 - C_{10} plane obtained by varying the MSSM parameters. The upper (lower) plot corresponds to solutions that satisfy the $b \rightarrow s\gamma$ experimental constraint with positive (negative) C_7 given in the text and the present bounds ($m_{H^+} > 80$ GeV, $\tilde{m}_t, m_{\chi^+}, \tilde{m}_\ell > 45$ GeV). The smaller areas limited by the short-dashed line correspond to the region of the MSSM parameter space that will survive an unsuccessful search for supersymmetry at the Tevatron and LEP 200 ($m_{H^+} > 150$ GeV, $m_{\tilde{l}}, m_{\chi^+}, m_{\tilde{l}} > 100$ GeV). (From [27].)

ery potential which goes beyond direct searches of SUSY particles at LEP-II and Tevatron, in case such searches remain negative.

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